Technical Notes

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Turbulent Mixing in Two-Dimensional Ducts with Transverse Jets

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Nomenclature

A = cross sectional area of duct
A_j = cross sectional area of jet nozzles
c' = rms concentration fluctuations
$\bar{c}$ = mean concentration
d = height of duct
J = momentum ratio of transverse jets to longitudinal flow
K = unknown dimensionless constant
t = Lagrangian time
U = speed of duct flow
V_j = speed of jet at the nozzle exit
x = downstream distance from nozzle exit
Ω = global rotation rate
ρ = duct fluid density
ρj = jet fluid density
τ = initial effective vortex rotation period

I. Introduction

TURBULENT mixing between two fluid streams in a confined duct is an important problem in technology. For example, the development of the pulsed chemical laser has led to the requirement of fast mixing, in a duct, of two streams containing different species.

In recent years the mixing rates have been measured in free, unconfined turbulent flows, where pure fluid is continually entrained into turbulent vorticies. The pure fluid is responsible for the extremum peaks in the probability density function (PDF)[1-3] of the concentration of an inert scalar. The number of extremum peaks is equal to the number of pure fluid streams. For example, the shear layer has two, and the farfield jet has one (Fig. 1).

The PDF also exhibits a more central, broad peak which represents the relatively well-mixed vortex cores.[3,4] It is of some interest to study this central peak in the PDF without the complicating presence of the extremum peaks and intervening valleys. The intervening valleys are associated with the diffusive interface between pure fluids,[4] analyzed by Marble as flame sheet[5] in the context of combustion. Both the flame sheet and pure fluid contributions to the PDF can be eliminated by cutting off the supply of pure fluid. A convenient way to achieve this end is to confine a vortex flow to a duct of some cross section. At a station downsteam of the point where both pure fluids are introduced into the duct, all elements of each pure fluid will have mixed with at least some elements of the opposite fluid, and the exremum peaks will disappear from the PDF; the only remaining feature would then be the central, broad peak. Hartung and Hiby[6] found this peak to be a gaussian for several tube geometries. This shape is expected in light of the central limit theorem, as noted by Dimotakis.[7] When the PDF becomes a gaussian, it is completely determined by two quantities: the location of its center $\bar{c}$; and the rms width c'. Their ratio, $c'/\bar{c}$, is a simple measure of the mixing for a confined vortex, where the extremum peaks have vanished.

A fundamental question concerns the universal nature of the stirring in such a vortex—cut off from pure fluid and constrained from growth by the duct. If the stirring is accomplished by inviscid, self-similar motions that are universal in turbulence, then one might expect a single parameter to exist which describes the contortions. The more the turbulence rearranges fluid elements at each global vortex revolution, the higher the probability that two fluid elements of widely differing initial concentration will find themselves adjacent to each other at the next time step, and thereby would mix by molecular diffusion. Hence, the width of the initial PDF peak should vary inversely with the contortion generated by the flow. A measure of contortion is the fractal dimension, which Mandelbrot[8] suggests is between 2.5–2.7 and equal to 8/3 for self-similar Kolmogorov turbulence. If the stirring characteristics of a vortex are universal, then the mixing rate should be insensitive to geometry. This Note reports the results of mixing experiments in a two-dimensional duct with transverse jets, and compares them with earlier experiments in a tube.

II. Flow Visualization

A lucite water tunnel was constructed with a test section of 15 cm span and 2.5 cm height d. The jet nozzles, located on both sides of the duct in a staggered geometry, with spanwise spacing equal to d between alternate nozzles, produced an array of counter-rotating vortices (Fig. 2). The aqueous streams flowing through the jet nozzles contained fluorescein. A transverse cross section of the duct downstream of the nozzles was illuminated with a sheet of light from an argon-ion laser. In this way the behavior of the injected fluid was revealed as the jet speed was varied. Typical Reynolds numbers in the water tunnel are 10^3 for the duct flow and 10^4 for the jets.

Figure 3 is a sequence of photographs illustrating the character of the flow as the jet speed is slowly reduced from an initially high value at a station $x/d = 2$. When the jet speed is high, the jet fluid is well mixed with the duct stream. But as the jet speed is reduced, distinct vortices become apparent. These can be traced from one image down to the next to observe the vortex trajectories. Finally, in the bottom image

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the counter-rotating vortex pairs are visible before they impinge on the wall opposite their nozzle of origin. Some nozzle fluid (injected at an earlier time) remains imbedded in the laminar wall boundary layer and has not yet been flushed out.

When a vortex approaches a wall, it induces opposite sign vorticity which may separate from the wall and roll up to form a small vortex of opposite sign to its parent. Two parent vortices with such daughters then combine to form a larger vortex. While the two-parent vortices rotate in the same sense and can thus naturally amalgamate into a larger vortex, the daughters cannot. Evidence of this conflict and rejection is apparent in Fig. 3. (Although not presented in this Note, flowfields similar to those in Fig. 3 have been observed at different x/d with a fixed jet speed.)

III. Concentration Measurements

The amplitude of the concentration fluctuations was measured in a duct of identical dimensions to the water tunnel previously discussed. Helium was injected through the jet nozzles into a duct stream of nitrogen, and the concentration of the resulting mixture was measured with a Brown-Rebollo aspirating probe.[9] Typical Reynolds numbers are about 10^3 for the duct flow and 10^4 for the transverse jets.

The rms amplitude of the concentration fluctuations c' is shown in Fig. 4 for several values of momentum ratio J, which is defined for incompressible flow as $J \equiv \Sigma \rho j V_j^2 A_j / \rho U^2 A$ where ρ, V, and A are the density, velocity, and area, respectively. The subscript j refers to the jet quantities; U is the initial nitrogen speed in the duct; and J represents the sum of the magnitude of the momenta of all the jets divided by the momentum of the incident duct stream.

The ratio $c'/\bar{c}$ was found to be independent of transverse or spanwise location over the entire cross secton of the duct, as long as the vorticies had grown to fill it. The lone exception occurred when an imperfect supply manifold injected more helium out of some nozzles than others. In that case, the mean concentration profile in the spanwise direction was maintained for the entire downstream length of the test section from the initial nonuniformity. Evidently, neighboring vortex cells mix very slowly. At their undulating boundary, c' was quite high when $\bar{c}$ differed between the neighboring vortices, as might be anticipated. But for uniform $\bar{c}$, c' was independent of probe location over any cross section.

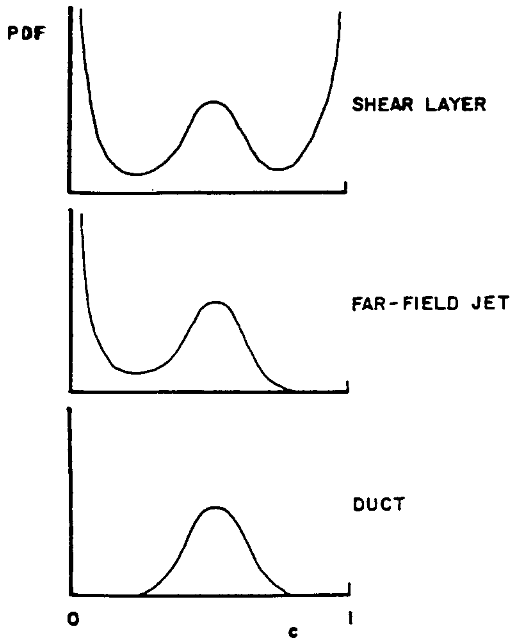

Fig. 1 Probability density function types.

Figure 4 indicates that the concentration fluctuations are reduced as the flow proceeds downstream and the momentum ratio J is increased. An empirical relationship from these results is

$$c'/\bar{c} \doteq 0.34 d/Jx \tag{1}$$

where d is the channel height and x is the downstream distance.

The duct measurements are compared with the earlier tube measurements in Fig. 5. Within the accuracy of the experiments, the results are nearly identical, suggesting a universal behavior insensitive to geometry.

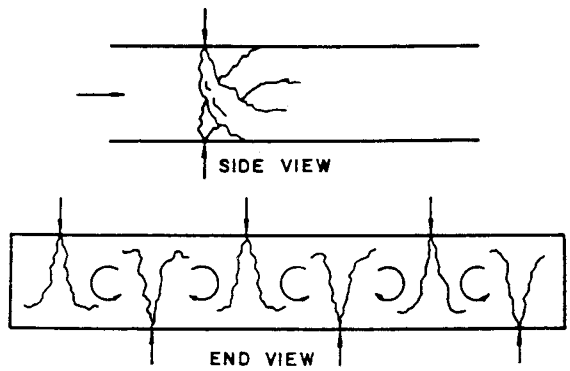

Fig. 2 Flow geometry.

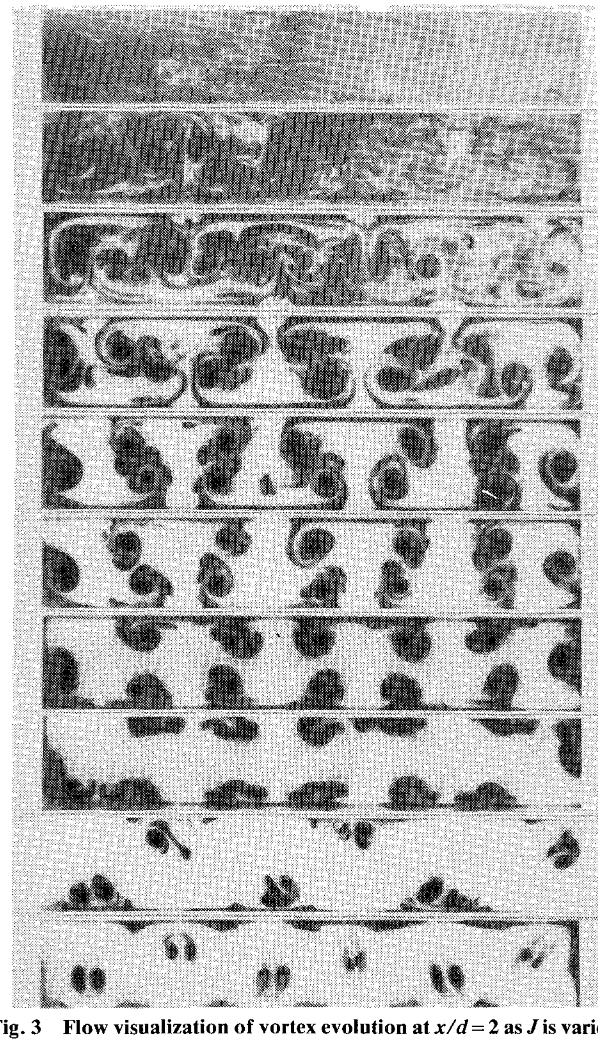

Fig. 3 Flow visualization of vortex evolution at $x/d = 2$ as J is varied.

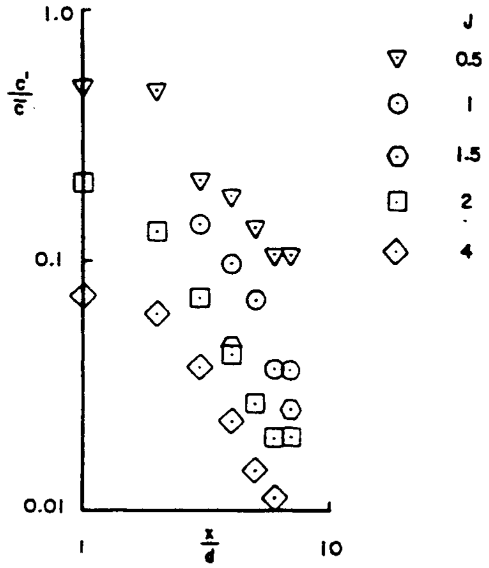

Fig. 4 Concentration fluctuations as a function of downstream distance for several values of momentum ratio J.

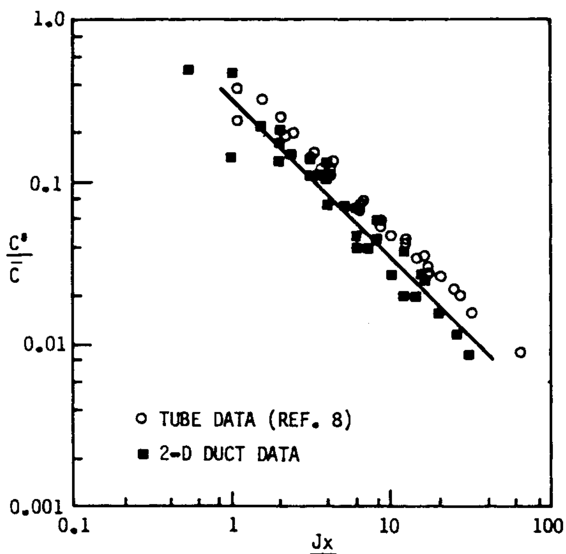

Fig. 5 Comparison of tube and rectangular duct results.

IV. Proposed Model

Thus, the simple theoretical model developed for the tube geometry[10] appears to have validity for the duct geometry. Briefly outlined, the effective rotation rate of the global vortices for rearrangement of fluid elements is taken by the model to be *precisely* equal to the inverse of the lagrangian age t of the vortices. At each rearrangement, the concentration fluctuations are presumed to decline to $1/e$ of their previous value. Hence

$$c'/\bar{c} = e^{-\int dt/t} = \tau/t$$

where τ is a characteristic scaling time. For this problem $\tau = Kd/JU$ is the initial effective vortex rotation period, where K is an unknown dimensionless coefficient. Taking the usual transformation $x = Ut$, the observations [Eq.(1)] are matched for $K = 0.34$.

Note that this simple model agrees with observation only if the effective rotation time (the e-folding time for the fluctuations) is precisely equal to the vortex age. For any self-similar flow, the rotation time must be proportional to the vortex age (there is no other available or imposed time scale). Here we make the stronger assertion that the proportionality factor is precisely unity. The only apparent alternative is to abandon the simple and attractive assumption of exponential decay with the number of vortex rotations. Evidence in support of exponential decay of concentration fluctuations in classical pipe flow is cited by Edwards et al.[10] Thus we are forced to conclude that the effective rearrangement time is just the vortex age, a remarkably simple result.

V. Other Models

A conventional, gradient diffusion model presents serious difficulties in these observations; there are no mean gradients in concentration on which to establish a mixing rate. Other approaches such as coalescence-dispersion[11] or PDF dynamics[3] are more physical. They mix parcels according to some algorithm at each time step. These approaches would be identical in principle to the proposed model if they make similar assumptions concerning the mixing frequency or rearrangement time.

VI. Conclusions

The amplitude of the concentration fluctuations in a two-dimensional duct downstream of an array of transverse jets was found to be nearly identical to that in a tube with a single jet. An empirical relation involving downstream distance, momentum ratio, and channel height describes the measurements. As in the earlier tube experiment, the results for the duct are consistent with a simple theoretical model which assumes both self-similar mixing and turbulence, where the effective rearrangement period of a vortex is precisely equal to its age.

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